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ELEMENTARY PARTICLE PHYSICS AND FIELD THEORY

NEW APPROACH TO THE QUANTUM TUNNELING PROCESS: CHARACTERISTIC TIMES FOR TRANSMISSION AND REFLECTION

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In [1] we have demonstrated that scattering of a quantum particle on a one-dimensional potential barrier should be considered as a combined process involving two alternative elementary transmission and reflection processes. For symmetric potential barriers, we have found solutions of the Schrödinger equation which describe the transmission and reflection processes in all stages of scattering. The present work studies time aspects of both processes. The local and asymptotic group tunneling times, dwell time, and Larmor tunneling time are determined for each process. Among these time characteristics, the group tunneling times should be considered as auxiliary. As to the dwell and Larmor tunneling times, they are the best estimates (of the expected values) of times the quantum particle in stationary and localized nonstationary states dwells in the barrier region. Moreover, the Larmor time is simply the dwell time averaged over the corresponding ensemble of particles. This characteristic can be measured experimentally and hence the suggested model of scattering can be verified.

INTRODUCTION

A new model of quantum particle scattering on a one-dimensional static potential barrier was proposed in [1]. According to this model, one-dimensional quantum scattering is a combined process involving two alternative elementary processes (transmission and reflection) macroscopically distinguishable in the final stage of scattering. For symmetric potential barriers, two solutions of the Schrödinger equation were obtained that describe both elementary processes in all stages of scattering. Their sum describes the state of the entire quantum ensemble of particles.

The tunneling model [1] provides a new approach to a solution of the problem of tunneling time posed in quantum mechanics almost since its origin (see [2–9]), but still unsolved. As is well known, this problem lies in the determination of the time of particle dwelling in the barrier region when scattering has been terminated. In this case, it is assumed that the source of particles and two detectors that register the transmitted and reflected particles are well away from the barrier. Thus, for the preset potential and initial particle state, the answer to the above-formulated problem should be unambiguous. In particular, the tunneling time should be independent of the details of measurements by the remote detectors.

We note that model [1] assumes separate timing of the transmitted and reflected particles in the barrier region. In both cases, the characteristic scattering times should be determined based on the wave functions for transmission and reflection, which comprise complete information on the behavior of each sub-ensemble in all stages of scattering. However, the timing procedure itself remains uncertain. As already pointed out in [1], no one of the known concepts of tunneling time is universal, since they endow the given process in this or that form by the nonlocality property that contradicts the principles of the special theory of relativity.

Among the known concepts, we have chosen the group tunneling time, dwell time, and Larmor tunneling time. Our problem is to revise these concepts based on the wave functions for transmission, $\psi_{\mathrm{tr}}(x,t)$, and reflection, $\psi_{\mathrm{ref}}(x,t)$, derived in [1]. In so doing, we assume that the statement of the scattering problem and the designations accepted in [1] remain in force in the present work.

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1. GROUP TUNNELING TIMES

As is well known, the expected particle velocity is equal to the velocity of the center of mass of the wave packet which describes the particle state. Therefore, the average time required for the particle to pass this or that distance can be estimated by timing the motion of the center of mass of the wave packet in this region. Therefore, we call the corresponding tunneling times the group times. We also consider the local and asymptotic group tunneling times.

Local group transmission and reflection times. Let t_1^{tr} and t_2^{tr} be moments of time such that

$$\frac{1}{\overline{T}} < \psi_{\mathrm{tr}}(x,t_1^{\mathrm{tr}}) \,|\, \hat{x} \,|\, \psi_{\mathrm{tr}}(x,t_1^{\mathrm{tr}}) >= a\,,\ \frac{1}{\overline{T}} < \psi_{\mathrm{tr}}(x,t_2^{\mathrm{tr}}) \,|\, \hat{x} \,|\, \psi_{\mathrm{tr}}(x,t_2^{\mathrm{tr}}) >= b\,, \tag{1}$$

where a and b are coordinates of the left and right boundaries of the potential barrier $V(x)$, $\overline{T}$ is the transmission coefficient ($<\psi_{\mathrm{tr}} \,|\, \psi_{\mathrm{tr}} >= \overline{T}$), and $\hat{x}$ is the operator of particle coordinate (see [1]). We define the local group time required for the particle to pass the barrier region $\tau_{\mathrm{tr}}^{\mathrm{gr}}$ as a difference $t_2^{\mathrm{tr}} - t_1^{\mathrm{tr}}$.

Similarly, we define the local group reflection time $\tau_{\mathrm{ref}}^{\mathrm{gr}}$ as a difference $t_{(+)} - t_{(-)}$, where the moments of time $t_{(+)}$ and $t_{(-)}$ are the maximum and minimum roots of the equation

$$\frac{1}{\overline{R}} < \psi_{\mathrm{ref}}(x,t_{(\pm)}) \,|\, \hat{x} \,|\, \psi_{\mathrm{ref}}(x,t_{(\pm)}) >= a\,. \tag{2}$$

Here $\overline{R}$ is the reflection coefficient ($<\psi_{\mathrm{ref}} \,|\, \psi_{\mathrm{ref}} >= \overline{R}$).

We note that consideration of the local group tunneling times does not solve the problem of tunneling time. The matter is that the position of the center of mass is poorly determined for sufficiently wide wave packets in the *x*-space and, as a consequence, the local group tunneling time gives only a very rough estimate of the dwell time of particle in the barrier region. This is clearly demonstrated by the example of reflection. Obviously, the local group reflection time is deliberately equal to zero if the wave packet width exceeds significantly the barrier width; the center of mass of the wide wave packet simply does not fall within the narrow barrier region.

Asymptotic group transmission and reflection times. We note that the potential barrier influences the particle not only when the center of mass of the wave packet falls within the barrier region. When scattering has been terminated, it is useful to consider the asymptotic group times of scattering that characterize the particle behavior in the wide interval $[a-L_1, b+L_2]$, where $L_1 >> l_0$, $L_2 >> l_0$, and l_0 is the half-width of the wave packet at the initial moment of time.

Obviously, in this case, instead of the exact wave functions, we can use the in- out-asymptotes in the *k*-representation (see expressions (6)–(9) of [1]). The total in-asymptote as well as the corresponding out-asymptote can be expressed as a sum of two wave packets:

$$f_{\mathrm{in}}(k,t) = f_{\mathrm{in}}^{\mathrm{tr}}(k,t) + f_{\mathrm{in}}^{\mathrm{ref}}(k,t)\,,$$

$$f_{\mathrm{in}}^{\mathrm{tr}}(k,t) = \sqrt{T(k)}A_{\mathrm{in}}(k)\exp\left[i\left(\lambda(k) - \frac{\pi}{2} - E(k)t/\hbar\right)\right], \tag{3}$$

$$f_{\mathrm{in}}^{\mathrm{ref}}(k,t) = \sqrt{R(k)}A_{\mathrm{in}}(k)\exp\left[i\left(\lambda(k) - E(k)t/\hbar\right)\right]. \tag{4}$$

Recall (see [1]) that $A_{\mathrm{in}}(k)$ is the Fourier transform of the in-state for the entire ensemble of particles.

For asymptotic expected values of the wave number we have

$$<k>_{\mathrm{in}}^{\mathrm{tr}} =<k>_{\mathrm{out}}^{\mathrm{tr}}, \quad <k>_{\mathrm{in}}^{\mathrm{ref}} = -<k>_{\mathrm{out}}^{\mathrm{ref}}, \quad \overline{T}\cdot<k>_{\mathrm{in}}^{\mathrm{tr}} + \overline{R}\cdot<k>_{\mathrm{in}}^{\mathrm{ref}} =<k>_{\mathrm{in}}^{\mathrm{full}} \equiv k_0\,.$$

In addition, it can be easily demonstrated that at small times,

$$<\hat{x}>_{\text{in}}^{\text{tr}}=\frac{\hbar t}{m}<\hat{k}>_{\text{in}}^{\text{tr}}-<\lambda'(k)>_{\text{in}}^{\text{tr}},\quad <\hat{x}>_{\text{in}}^{\text{ref}}=\frac{\hbar t}{m}<\hat{k}>_{\text{in}}^{\text{ref}}-<\lambda'(k)>_{\text{in}}^{\text{ref}}. \tag{5}$$

Hereinafter, the prime denotes the derivative with respect to k, and the angular brackets denote average values for the corresponding quantum state.

As follows from Eqs. (5), the most probable starting point coordinates for both sub-ensembles are determined by expressions

$$<\hat{x}>_{\text{start}}^{\text{tr}}=-<\lambda'(k)>_{\text{in}}^{\text{tr}},\quad <\hat{x}>_{\text{start}}^{\text{ref}}=-<\lambda'(k)>_{\text{in}}^{\text{ref}}. \tag{6}$$

Thus, in the given statement of the scattering problem, the transmitted and reflected particles start, on average, from points $<\hat{x}>_{\text{start}}^{\text{tr}}$ and $<\hat{x}>_{\text{start}}^{\text{ref}}$, respectively, rather than from the origin of coordinates, which is the case for the entire ensemble of particles (see [1]). It is important to emphasize once again that these quantities are initial values of functions $<\hat{x}>_{\text{in}}^{\text{tr}}(t)$ and $<\hat{x}>_{\text{in}}^{\text{ref}}(t)$ which have the status of expected values of the operator $\hat{x}$. As for $<\hat{x}>_{\text{in}}(t)$, this average value, obtained for the entire ensemble of particles for the examined problem, cannot be interpreted as the most probable value of the particle coordinate (see [1]).

The average value of the particle coordinate for both sub-ensembles after scattering is determined by expressions

$$<\hat{x}>_{\text{out}}^{\text{tr}}=\frac{\hbar t}{m}<\hat{k}>_{\text{out}}^{\text{tr}}-<J'(k)>_{\text{out}}^{\text{tr}}+d,\quad <\hat{x}>_{\text{out}}^{\text{ref}}=\frac{\hbar t}{m}<\hat{k}>_{\text{out}}^{\text{ref}}-<J'(k)-F'(k)>_{\text{out}}^{\text{ref}}+2a. \tag{7}$$

Now we can find the asymptotic group transmission and reflection times for the interval $[a-L_1,b+L_2]$. Let t_1^{tr} and t_2^{tr} be moments of time such that

$$<\hat{x}>_{\text{in}}^{\text{tr}}(t_1^{\text{tr}})=a-L_1,\quad <\hat{x}>_{\text{out}}^{\text{tr}}(t_2^{\text{tr}})=b+L_2.$$

With allowance for Eqs. (5) and (7), we obtain that the time required for the particle to pass this interval is

$$\tau_{\text{tr}}(a-L_1,b+L_2)\equiv t_2^{\text{tr}}-t_1^{\text{tr}}=\frac{m}{\hbar<k>_{\text{in}}^{\text{tr}}}\left(<J'>_{\text{out}}^{\text{tr}}-<\lambda'>_{\text{in}}^{\text{tr}}+L_1+L_2\right).$$

Similarly, let t_1^{tr} and t_2^{tr} be moments of time such that

$$<\hat{x}>_{\text{in}}^{\text{ref}}(t_1^{\text{ref}})=a-L_1,\quad <\hat{x}>_{\text{out}}^{\text{ref}}(t_2^{\text{ref}})=a-L_1.$$

Then for the reflection time $\tau_{\text{ref}}(a-L_1,b+L_2)$, where $\tau_{\text{ref}}(a-L_1,b+L_2)\equiv t_2^{\text{ref}}-t_1^{\text{ref}}$, with allowance for Eqs. (5) and (7), we obtain

$$\tau_{\text{ref}}(a-L_1,b+L_2)=\frac{m}{\hbar<k>_{\text{in}}^{\text{ref}}}\left(<J'-F'>_{\text{out}}^{\text{ref}}-<\lambda'>_{\text{in}}^{\text{ref}}+2L_1\right).$$

The quantities $\tau_{\text{tr}}^{\text{as}}$ and $\tau_{\text{ref}}^{\text{as}}$, where $\tau_{\text{tr}}^{\text{as}}=\tau_{\text{tr}}(a,b)$ and $\tau_{\text{ref}}^{\text{as}}=\tau_{\text{ref}}(a,b)$, are called the asymptotic group tunneling transmission and reflection times, respectively:

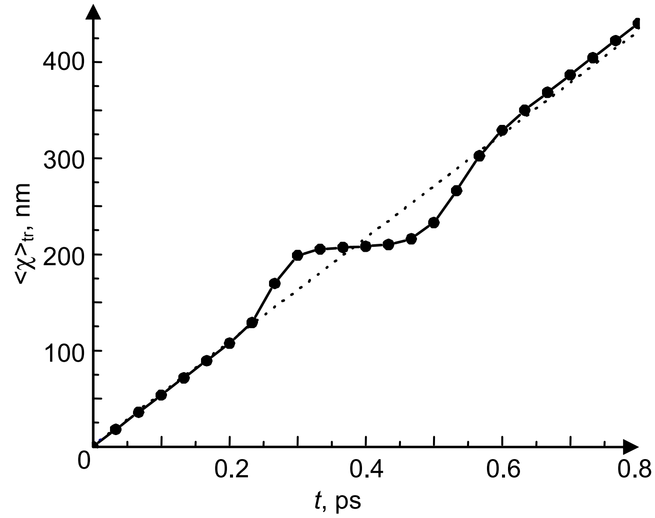

Fig. 1. Time variations of the position of the center of mass of the wave packet for transmission.

$$\tau_{\mathrm{tr}}^{\mathrm{as}} = \frac{m}{\hbar < k >_{\mathrm{in}}^{\mathrm{tr}}} \left(< J' >_{\mathrm{out}}^{\mathrm{tr}} - < \lambda' >_{\mathrm{in}}^{\mathrm{tr}} \right), \tag{8}$$

$$\tau_{\mathrm{ref}}^{\mathrm{as}} = \frac{m}{\hbar < k >_{\mathrm{in}}^{\mathrm{ref}}} \left(< J' - F' >_{\mathrm{out}}^{\mathrm{ref}} - < \lambda' >_{\mathrm{in}}^{\mathrm{ref}} \right). \tag{9}$$

Furthermore, the parameters $d_{\mathrm{eff}}^{\mathrm{tr}}$ and $d_{\mathrm{eff}}^{\mathrm{ref}}$, where $d_{\mathrm{eff}}^{\mathrm{tr}} = < J' >_{\mathrm{out}}^{\mathrm{tr}} - < \lambda' >_{\mathrm{in}}^{\mathrm{tr}}$ and $d_{\mathrm{eff}}^{\mathrm{ref}} = < J' - F' >_{\mathrm{out}}^{\mathrm{ref}} - < \lambda' >_{\mathrm{in}}^{\mathrm{ref}}$, are called the effective barrier widths for transmission and reflection, respectively.

We note that $\tau_{\mathrm{tr}}^{\mathrm{as}}$ and $\tau_{\mathrm{ref}}^{\mathrm{as}}$ values, unlike the local group times $\tau_{\mathrm{tr}}^{\mathrm{gr}}$ and $\tau_{\mathrm{ref}}^{\mathrm{gr}}$, can be negative, since the asymptotic characteristics do not estimate the dwell time of particle in the barrier region.

Figure 1 shows the dependence $< \hat{x} >_{\mathrm{tr}} (t)$. Calculations were carried out for particle tunneling through a rectangular potential barrier from the initial state described by the Gaussian wave packet with $a = 200$ nm, $b = 215$ nm, $V_0 = 0.2$ eV, and $l_0 = 10$ nm; the average particle energy at the initial moment of time was 0.05 eV.

The difference between the local and asymptotic group transmission times is well seen from Fig. 1. Whereas $\tau_{\mathrm{tr}}^{\mathrm{gr}}$ specifies the dwell time of the center of mass of the wave packet in the barrier region, $\tau_{\mathrm{tr}}^{\mathrm{as}}$ serves as a measure of the barrier influence on the motion of the center of mass during scattering. The quantity $\tau_{\mathrm{tr}}^{\mathrm{as}} - \tau_{\mathrm{free}}$, where $\tau_{\mathrm{free}} = md / \hbar k_0$, is the most probable value of the time delay (or advance) for the particle that has been passed through the barrier in comparison with its free motion. In this case, $\tau_{\mathrm{tr}}^{\mathrm{gr}} \approx 0.155$ ps, $\tau_{\mathrm{tr}}^{\mathrm{as}} \approx 0.01$ ps, and $\tau_{\mathrm{free}} \approx 0.025$ ps. Thus, the influence of the opaque barrier on the particle is complex in character. Though the barrier decelerates significantly the particle motion in the barrier region, the resultant influence of the barrier appears accelerating. The particle that has been passed through the barrier moves, on average, with an advance compared to its free motion.

Average starting points and asymptotic group times for a rectangular potential barrier. We now consider the stationary problem of scattering on rectangular potential barriers. Since $F'(k) \equiv 0$ for symmetric barriers, the effective barrier widths and the starting points for the ensembles of transmitted and reflected particles with preset energies will be identical:

$$\tau_{\mathrm{tr}}^{\mathrm{as}} = \tau_{\mathrm{ref}}^{\mathrm{as}} = \frac{m d_{\mathrm{eff}}(k_0)}{\hbar k_0}, \quad x_{\mathrm{tr}}^{\mathrm{as}} = x_{\mathrm{ref}}^{\mathrm{as}} = x_{\mathrm{start}} = -\lambda'(k)\big|_{k=k_0}.$$

Using expressions for the real tunneling parameters $T(k)$ and $J(k)$ and their derivatives (see [18, 23]), we can demonstrate that above the barrier ($E \le V_0$), these parameters are

$$d_{\text{eff}}(k) = \frac{4}{\kappa} \frac{\left[k^2 + \kappa_0^2 \sinh^2(\kappa d/2)\right]\left[\kappa_0^2 \sinh(\kappa d) - k^2 \kappa d\right]}{4k^2\kappa^2 + \kappa_0^4 \sinh^2(\kappa d)}, \tag{10}$$

$$x_{\text{start}}(k) = -2\frac{\kappa_0^2}{\kappa} \frac{\left(\kappa^2 - k^2\right)\sinh(\kappa d) + k^2 \kappa d \cosh(\kappa d)}{4k^2\kappa^2 + \kappa_0^4 \sinh^2(\kappa d)};$$

below the barrier ($E > V_0$), they are

$$d_{\text{eff}}(k) = \frac{4}{\kappa} \frac{\left[k^2 - \beta\kappa_0^2 \sin^2(\kappa d/2)\right]\left[k^2 \kappa d - \beta\kappa_0^2 \sin(\kappa d)\right]}{4k^2\kappa^2 + \kappa_0^4 \sin^2(\kappa d)}, \tag{11}$$

$$x_{\text{start}}(k) = -2\beta\frac{\kappa_0^2}{\kappa} \frac{\left(\kappa^2 + k^2\right)\sin(\kappa d) - k^2 \kappa d \cos(\kappa d)}{4k^2\kappa^2 + \kappa_0^4 \sin^2(\kappa d)},$$

where $\kappa_0 = \sqrt{2m\,|V_0|/\hbar^2}$ and $\beta = 1$ for $V_0 > 0$; otherwise, $\beta = -1$.

We note that $d_{\text{eff}} \to d$ and $x_{\text{start}} \to 0$ when $k \to \infty$. This property guarantees that averaging of the starting points separately over the transmitted and reflected particles and over the entire ensemble will give the same result when $l_0 \to 0$, that is, the origin of coordinates (in the examined statement of the problem).

Below the barrier ($E < V_0$), we have $d_{\text{eff}} \approx 2/\kappa$ and $x_{\text{start}} \approx 0$ for the particle that has passed through wide barriers ($\kappa d >> 1$). Thus, the asymptotic tunneling time is saturated with increase in the opaque barrier width. However, this does not mean the infinite growth of the effective tunneling velocity, since the asymptotic tunneling time describes the influence of the barrier on the particle during complete scattering. This means that this characteristic refers to the spatial interval considerably exceeding the barrier region.

It should be noted further that $d_{\text{eff}} = 0$ for the δ-potential ($V(x) = W \cdot \delta(x-a)$). In this case, each sub-ensemble, that is, both transmitted and reflected particles, starts, on average, from the same points $x_{\text{start}}(k) = -2m\hbar^2 W/(\hbar^4 k^2 + m^2 W^2)$.

2. DWELL TIME

Let us consider now the stationary scattering problem. It describes the limiting case of scattering of wide wave packets when the local group tunneling time is inapplicable for estimation of the dwell time of particle in the barrier region. In this case, the dwell time concept [15] is more suitable.

Dwell time for transmission. We note that in the case of transmission, the probability flux density I_{tr} can be written as $v_{\text{tr}} \cdot |\psi_{\text{tr}}(x,k)|^2$, where v_{tr} is the velocity of an infinitesimally small element of the flux. Thus, we have $v_{\text{tr}}(x,k) = I_{\text{tr}}(k) \cdot |\psi_{\text{tr}}(x,k)|^{-2}$.

Since the time required for the particle to pass the distance $[x, x+dx]$ is equal to $dx/v_{\text{tr}}(x,k)$, the total time required for the given flux element to pass through the barrier is given by the expression

$$\tau_{\text{dwell}}^{\text{tr}}(k) = \frac{1}{I_{\text{tr}}} \int_a^b |\psi_{\text{tr}}(x,k)|^2 dx. \tag{12}$$

It can be easily demonstrated that for a wide rectangular barrier and transmission under the barrier, the velocity of the given flux element decreases exponentially as the barrier center is approached. In this case, $|\psi_{\mathrm{tr}}(a,k)|=|\psi_{\mathrm{tr}}(b,k)|=\sqrt{T(k)}$; at the same time, $|\psi_{\mathrm{tr}}(x_c,k)|\propto\sqrt{T}\cdot\exp(\kappa d/2)$. This causes the dwell time of the particle in the opaque barrier region to increase exponentially with increase in the barrier width. Indeed, with allowance for Eqs. (21) and (22) (see [1]), for $\psi_{\mathrm{tr}}(x,k)$ of the rectangular barrier we have

$$\tau_{\mathrm{dwell}}^{\mathrm{tr}}=\frac{m}{2\hbar k\kappa^3}\left[\left(\kappa^2-k^2\right)\kappa d+\kappa_0^2\sinh(\kappa d)\right],\quad E<V_0, \tag{13}$$

$$\tau_{\mathrm{dwell}}^{\mathrm{tr}}=\frac{m}{2\hbar k\kappa^3}\left[\left(\kappa^2+k^2\right)\kappa d-\beta\kappa_0^2\sin(\kappa d)\right],\quad E\geq V_0. \tag{14}$$

Dwell time for reflection. We note that the probability flux density for the wave function $\psi_{\mathrm{ref}}(x,k)$ is equal to zero. Therefore, the arguments we used to introduce the dwell time for transmission are unsuitable here. By analogy with [15], we define the dwell time for reflection with the help of the expression

$$\tau_{\mathrm{dwell}}^{\mathrm{ref}}(k)=\frac{1}{I_{\mathrm{ref}}}\int_a^{x_c}\left|\psi_{\mathrm{ref}}(x,k)\right|^2dx, \tag{15}$$

where $I_{\mathrm{ref}}=\hbar kR/m$ is the probability flux density of the incident wave in the case of reflection. For the rectangular barrier, after substitution of Eq. (19) into Eq. (15) (see [1]), we obtain

$$\tau_{\mathrm{dwell}}^{\mathrm{ref}}=\frac{mk}{\hbar\kappa}\frac{\sinh(\kappa d)-\kappa d}{\kappa^2+\kappa_0^2\sinh^2(\kappa d/2)},\quad E<V_0, \tag{16}$$

$$\tau_{\mathrm{dwell}}^{\mathrm{ref}}=\frac{mk}{\hbar\kappa}\frac{\kappa d-\sin(\kappa d)}{\kappa^2+\beta\kappa_0^2\sin^2(\kappa d/2)},\quad E\geq V_0. \tag{17}$$

3. THE LARMOR TUNNELING TIME

We note that the concepts of the group tunneling time considered above and of the dwell time (they complement each other) should be considered only as auxiliary. First, both rules of timing of the tunneling process have limited applicability. Second and more important, they do not provide a direct method of measuring the dwell time of the particle in the barrier region. Our next step is to demonstrate that both problems are successfully solved within the framework of the concept of the Larmor tunneling time suggested in [21] and subsequently developed in [22, 15].

It should be noted, however, that the well-known definitions of the Larmor tunneling time have serious disadvantages since they are based on the amplitude and phase of the outgoing (transmitted and reflected) waves. At the same time, to describe the influence of the magnetic field on the spin of particle in the barrier region, the wave function exactly in this region must be known. In this connection, the concept of the Larmor tunneling time should be revised with allowance for this circumstance.

Let us assume that a particle with a spin of 1/2 (for example, an electron) is incident from the left on the potential barrier $V(x)$ (see [1]) in the region of which a weak uniform magnetic field directed along the *OZ* axis is switched on. Let the in-state of the particle be described by the spinor

$$\Psi_{\mathrm{in}}(x)=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}\psi_{\mathrm{in}}(x), \tag{18}$$

where the function $\psi_{\text{in}}(x)$ satisfies conditions (1) (see [1]). In fact, we assume that at the initial moment of time $t=0$ there is a mixture of two particle ensembles that differ only by the spin directions: the particle spins in the first ensemble are directed upwards (parallel to the magnetic field), and in the second ensemble they are directed downwards. Spinor (18) is an eigenvector of the matrix σ_x and its eigenvalue is 1. Hereinafter, σ_x, σ_y, and σ_z are the Pauli matrices.

For the electrons with upward (downward) spins, the potential barrier height effectively decreases (increases) by $\hbar\omega_{\text{L}}/2$, where ω_{L} is the Larmor spin precession frequency $\omega_{\text{L}} = 2\mu B/\hbar$, B is the magnetic field strength, and μ is the electron magnetic moment. The corresponding Hamiltonian is written in the form

$$\hat{H} = \left[\frac{\hat{p}^2}{2m} + V(x)\right]^{\bar{\bar{1}}} - \frac{\hbar\omega_{\text{L}}}{2}\sigma_z, \quad x \in [a,b]; \quad \hat{H} = \frac{\hat{p}^2}{2m}\cdot \bar{\bar{1}}, \quad x \notin [a,b]. \tag{19}$$

For $t>0$, the states of particles with upward and downward spins differ due to the action of the magnetic field. The probability of barrier passage is different for them. Let the particle state at the moment of time *t* be described with the spinor

$$\Psi_{\text{full}}(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} \psi_{\text{full}}^{(\uparrow)}(x,t) \\ \psi_{\text{full}}^{(\downarrow)}(x,t) \end{pmatrix}. \tag{20}$$

According to [1], each of the spinor components can be unambiguously expressed as a coherent superposition of two wave fields that describe the transmission and reflection processes:

$$\psi_{\text{full}}^{(\uparrow)}(x,t) = \psi_{\text{tr}}^{(\uparrow)}(x,t) + \psi_{\text{ref}}^{(\uparrow)}(x,t), \quad \psi_{\text{full}}^{(\downarrow)}(x,t) = \psi_{\text{tr}}^{(\downarrow)}(x,t) + \psi_{\text{ref}}^{(\downarrow)}(x,t), \tag{21}$$

(we note that $\psi_{\text{ref}}^{(\uparrow\downarrow)}(x,t) \equiv 0$ for $x \ge x_c$). As a consequence, the same expression can be written for spinor (20): $\Psi_{\text{full}}(x,t) = \Psi_{\text{tr}}(x,t) + \Psi_{\text{ref}}(x,t)$.

Below we assume that the wave functions for transmission and reflection have already been known. Here it is important to emphasize that

$$< \psi_{\text{full}}^{(\uparrow\downarrow)}(x,t) \,|\, \psi_{\text{full}}^{(\uparrow\downarrow)}(x,t) > = T^{(\uparrow\downarrow)} + R^{(\uparrow\downarrow)} = 1, \tag{22}$$

$$T^{(\uparrow\downarrow)} = < \psi_{\text{tr}}^{(\uparrow\downarrow)}(x,t) \,|\, \psi_{\text{tr}}^{(\uparrow\downarrow)}(x,t) > = \text{const}, \quad R^{(\uparrow\downarrow)} = < \psi_{\text{ref}}^{(\uparrow\downarrow)}(x,t) \,|\, \psi_{\text{ref}}^{(\uparrow\downarrow)}(x,t) > = \text{const},$$

$T^{(\uparrow\downarrow)}$ and $R^{(\uparrow\downarrow)}$ are the transmission and reflection coefficients, respectively, for particles with upward $(\uparrow)$ and downward spins $(\downarrow)$. Furthermore, let $T = (T^{(\uparrow)} + T^{(\downarrow)})/2$ and $R = (R^{(\uparrow)} + R^{(\downarrow)})/2$.

Time variations of particle spin polarization. To study time variations of the spin direction, we must determine the expected values of the spin projections S_x, S_y, and S_z. For an arbitrary moment of time *t*, we obtain

$$\langle S_x \rangle \equiv \frac{\hbar}{2}\sin(\theta_{\text{full}})\cos(\phi_{\text{full}}) = \hbar \cdot \text{Re}\left(< \psi_{\text{full}}^{(\uparrow)} \,|\, \psi_{\text{full}}^{(\downarrow)} >\right),$$

$$\langle S_y \rangle \equiv \frac{\hbar}{2}\sin(\theta_{\text{full}})\sin(\phi_{\text{full}}) = \hbar \cdot \text{Im}\left(< \psi_{\text{full}}^{(\uparrow)} \,|\, \psi_{\text{full}}^{(\downarrow)} >\right), \tag{23}$$

$$\langle S_z \rangle \equiv \frac{\hbar}{2}\cos(\theta_{\text{full}}) = \frac{\hbar}{2}\left(< \psi_{\text{full}}^{(\uparrow)} \,|\, \psi_{\text{full}}^{(\uparrow)} > - < \psi_{\text{full}}^{(\downarrow)} \,|\, \psi_{\text{full}}^{(\downarrow)} >\right).$$

Analogous expressions can be written for the sub-ensembles of the transmitted and reflected particles.

We note that $\theta_{\text{full}} = \pi/2$ and $\phi_{\text{full}} = 0$ for spinor (20) at $t = 0$. However, for transmission and reflection at $t = 0$ we have

$$\phi^{(0)}_{\text{tr,ref}} = \arctan\left(\frac{\text{Im}\left\langle \psi^{(\uparrow)}_{\text{tr,ref}}(x,0) \,|\, \psi^{(\downarrow)}_{\text{tr,ref}}(x,0)\right\rangle}{\text{Re}\left\langle \psi^{(\uparrow)}_{\text{tr,ref}}(x,0) \,|\, \psi^{(\downarrow)}_{\text{tr,ref}}(x,0)\right\rangle}\right), \tag{24}$$

$$\theta^{(0)}_{\text{tr,ref}} = \arccos\left(\left\langle \psi^{(\uparrow)}_{\text{tr,ref}}(x,0) \,|\, \psi^{(\uparrow)}_{\text{tr,ref}}(x,0)\right\rangle - \left\langle \psi^{(\downarrow)}_{\text{tr,ref}}(x,0) \,|\, \psi^{(\downarrow)}_{\text{tr,ref}}(x,0)\right\rangle\right).$$

The norms of all wave functions in Eqs. (24) are kept unchanged. Therefore, we have $\theta_{\text{tr}}(t) \equiv \theta^{(0)}_{\text{tr}}$ and $\theta_{\text{ref}}(t) \equiv \theta^{(0)}_{\text{ref}}$ at any moment of time *t*. It can be easily demonstrated that

$$< \hat{S}_z >_{\text{tr}} (t) = \hbar \frac{T^{(\uparrow)} - T^{(\downarrow)}}{T^{(\uparrow)} + T^{(\downarrow)}}, \quad < \hat{S}_z >_{\text{ref}} (t) = \hbar \frac{R^{(\uparrow)} - R^{(\downarrow)}}{R^{(\uparrow)} + R^{(\downarrow)}}. \tag{25}$$

Hence it follows that since the operator $\hat{S}_z$ commutes with Hamiltonian (19), the given spin projection, on average, is kept unchanged during scattering for both macroscopically distinguishable processes. Thus, the angles $\theta_{\text{tr}}(t)$ and $\theta_{\text{ref}}(t)$ cannot measure the dwell time of the particle in the region of action of the magnetic field when $t \to \infty$.

The Larmor spin precession in an infinitesimal magnetic field. Our next step is to use the spin precession as a clock to determine the dwell time of the particle in the barrier region. In accordance with [15], we assume that the magnetic field strength is as small as is wished (we note that in all other respects our approach differs essentially from that used in [15]).

First we emphasize once again that the angles $\theta_{\text{tr}}(t)$ and $\theta_{\text{ref}}(t)$ are kept unchanged during particle scattering (they are equal to $\theta^{(0)}_{\text{tr}}$ and $\theta^{(0)}_{\text{ref}}$, respectively), and hence they cannot be used for timing the particle motion in the barrier region. For the infinitesimal magnetic field and rectangular potential barrier, these parameters are determined by expressions $\theta^{(0)}_{\text{tr}} = \frac{\pi}{2} - \omega_{\text{L}}\tau_z$ and $\theta^{(0)}_{\text{ref}} = \frac{\pi}{2} + \omega_{\text{L}}\tau_z$, where

$$\tau_z = \frac{m\kappa_0^2}{\hbar\kappa^2} \frac{\left(\kappa^2 - k^2\right)\sinh(\kappa d) + \kappa_0^2 \kappa d \cosh(\kappa d)}{4k^2\kappa^2 + \kappa_0^4 \sinh^2(\kappa d)} \sinh(\kappa d), \quad E < V_0, \tag{26}$$

$$\tau_z = \frac{m\kappa_0^2}{\hbar\kappa^2} \frac{\kappa_0^2 \kappa d \cos(\kappa d) - \beta\left(\kappa^2 + k^2\right)\sin(\kappa d)}{4k^2\kappa^2 + \kappa_0^4 \sin^2(\kappa d)} \sin(\kappa d), \quad E \geq V_0. \tag{27}$$

We note that τ_z in [15] [see Eq. (2.20a)] is treated as the characteristic tunneling time. However, in our approach this parameter is not related to the duration of the tunneling process.

Proceeding to the determination of the tunneling time, we note once again that the hands of the clock used for this purpose do not generally point to zero when the particle enters the barrier (that is, the angles $\phi^{(0)}_{\text{tr}}$ and $\phi^{(0)}_{\text{ref}}$ are not equal to zero). It can be easily demonstrated that $\phi^{(0)}_{\text{tr}} = \omega_{\text{L}}\tau_0$ and $\phi^{(0)}_{\text{ref}} = -\omega_{\text{L}}\tau_0$ for the rectangular barrier and infinitesimal magnetic field, where

$$\tau_0 = \frac{2mk}{\hbar\kappa} \frac{\left(\kappa^2 - k^2\right)\sinh(\kappa d) + \kappa_0^2 \kappa d \cosh(\kappa d)}{4k^2\kappa^2 + \kappa_0^4 \sinh^2(\kappa d)}, \quad E < V_0, \tag{28}$$

$$\tau_0 = \frac{2mk}{\hbar\kappa}\frac{\beta\kappa_0^2\kappa d\cos(\kappa d) - \left(\kappa^2 + k^2\right)\sin(\kappa d)}{4k^2\kappa^2 + \kappa_0^4\sin^2(\kappa d)}, \quad E \geq V_0 . \tag{29}$$

We now find the derivatives $d\phi_{\mathrm{tr}}(t)/dt$ and $d\phi_{\mathrm{ref}}(t)/dt$. To this end, we take advantage of the Ehrenfest equations for the expected values of particle spin projections:

$$\frac{d < \hat{S}_x >_{\mathrm{tr,ref}}}{dt} = -\hbar\omega_{\mathrm{L}} \int_a^{x_{\max}} \mathrm{Im}\left[\left(\psi_{\mathrm{tr,ref}}^{(\uparrow)}(x,t)\right)^* \psi_{\mathrm{tr,ref}}^{(\downarrow)}(x,t)\right] dx , \tag{30}$$

$$\frac{d < \hat{S}_y >_{\mathrm{tr,ref}}}{dt} = -\hbar\omega_{\mathrm{L}} \int_a^{x_{\max}} \mathrm{Re}\left[\left(\psi_{\mathrm{tr,ref}}^{(\uparrow)}(x,t)\right)^* \psi_{\mathrm{tr,ref}}^{(\downarrow)}(x,t)\right] dx ,$$

where $x_{\max} = b$ for transmission and $x_{\max} = x_c$ for reflection.

We note that $\phi_{\mathrm{tr,ref}} = \arctan\left(< \hat{S}_y >_{\mathrm{tr,ref}} / < \hat{S}_x >_{\mathrm{tr,ref}}\right)$. Hence it follows that

$$\frac{d\phi_{\mathrm{tr,ref}}}{dt} = \frac{< \hat{S}_x >_{\mathrm{tr,ref}} \dfrac{d < \hat{S}_y >_{\mathrm{tr,ref}}}{dt} - < \hat{S}_y >_{\mathrm{tr,ref}} \dfrac{d < \hat{S}_x >_{\mathrm{tr,ref}}}{dt}}{< \hat{S}_x >_{\mathrm{tr,ref}}^2 + < \hat{S}_y >_{\mathrm{tr,ref}}^2} .$$

However, for initial condition (18) and weak magnetic field, when inequalities $\left|< \hat{S}_y >_{\mathrm{tr}}\right| << \left|< \hat{S}_x >_{\mathrm{tr}}\right|$ and $\left|< \hat{S}_y >_{\mathrm{ref}}\right| << \left|< \hat{S}_x >_{\mathrm{ref}}\right|$ are valid, expressions for $d\phi_{\mathrm{tr}}(t)/dt$ and $d\phi_{\mathrm{ref}}(t)/dt$ are significantly simplified:

$$\frac{d\phi_{\mathrm{tr}}}{dt} = \frac{1}{< \hat{S}_x >_{\mathrm{tr}}}\frac{d < \hat{S}_y >_{\mathrm{tr}}}{dt}, \quad \frac{d\phi_{\mathrm{ref}}}{dt} = \frac{1}{< \hat{S}_x >_{\mathrm{ref}}}\frac{d < \hat{S}_y >_{\mathrm{ref}}}{dt} .$$

Taking into account expressions for spin projections and their derivatives [see Eqs. (23) and (30)], we obtain

$$\frac{d\phi_{\mathrm{tr}}}{dt} \approx \omega_{\mathrm{L}} \frac{\int_a^b \mathrm{Re}\left[\left(\psi_{\mathrm{tr}}^{(\uparrow)}(x,t)\right)^* \psi_{\mathrm{tr}}^{(\downarrow)}(x,t)\right] dx}{\int_{-\infty}^{\infty} \mathrm{Re}\left[\left(\psi_{\mathrm{tr}}^{(\uparrow)}(x,t)\right)^* \psi_{\mathrm{tr}}^{(\downarrow)}(x,t)\right] dx}, \quad \frac{d\phi_{\mathrm{ref}}}{dt} \approx \omega_{\mathrm{L}} \frac{\int_a^{x_c} \mathrm{Re}\left[\left(\psi_{\mathrm{ref}}^{(\uparrow)}(x,t)\right)^* \psi_{\mathrm{ref}}^{(\downarrow)}(x,t)\right] dx}{\int_{-\infty}^{x_c} \mathrm{Re}\left[\left(\psi_{\mathrm{ref}}^{(\uparrow)}(x,t)\right)^* \psi_{\mathrm{ref}}^{(\downarrow)}(x,t)\right] dx} .$$

If we consider that $\psi_{\mathrm{tr,ref}}^{(\uparrow)}(x,t) = \psi_{\mathrm{tr,ref}}^{(\downarrow)}(x,t) = \psi_{\mathrm{tr,ref}}(x,t)$ for the zero order of smallness in ω_{L}, we obtain

$$\frac{d\phi_{\mathrm{tr}}}{dt} \approx \frac{\omega_{\mathrm{L}}}{T}\int_a^b \left|\psi_{\mathrm{tr}}(x,t)\right|^2 dx > 0, \quad \frac{d\phi_{\mathrm{ref}}}{dt} \approx \frac{\omega_L}{R}\int_a^{x_c} \left|\psi_{\mathrm{ref}}(x,t)\right|^2 dx > 0 .$$

As can be seen, the clock operates properly, since the hands rotate in the same directions.

Recall now that in our statement of the problem, particles interact neither with the potential barrier, nor with the magnetic field at the initial and final moments of time. This means that the total spin precession angles $\Delta\phi_{\mathrm{tr}}$ and $\Delta\phi_{\mathrm{ref}}$ by the time the scattering process terminates can be written as

$$\Delta\phi_{\rm tr}=\frac{\omega_L}{T}\int_{-\infty}^{\infty}dt\int_a^b dx\left|\psi_{\rm tr}(x,t)\right|^2,\quad \Delta\phi_{\rm ref}=\frac{\omega_L}{R}\int_{-\infty}^{\infty}dt\int_a^{x_c} dx\left|\psi_{\rm ref}(x,t)\right|^2 . \tag{31}$$

On the other hand, these angles can be expressed as follows:

$$\Delta\phi_{\rm tr}=\omega_{\rm L}\tau_{\rm tr}^{\rm L},\quad \Delta\phi_{\rm ref}=\omega_{\rm L}\tau_{\rm ref}^{\rm L}, \tag{32}$$

where $\tau_{\rm tr}^{\rm L}$ and $\tau_{\rm ref}^{\rm L}$ are the sought-after Larmor tunneling times. Comparing Eqs. (31) and (32), we finally obtain

$$\tau_{\rm tr}^{\rm L}=\frac{1}{T}\int_{-\infty}^{\infty}dt\int_a^b dx\left|\psi_{\rm tr}(x,t)\right|^2,\quad \tau_{\rm ref}^{\rm L}=\frac{1}{R}\int_{-\infty}^{\infty}dt\int_a^{x_c} dx\left|\psi_{\rm ref}(x,t)\right|^2 . \tag{33}$$

Relation between the Larmor tunneling and dwell times. We note that the form of Eqs. (33) for the Larmor tunneling times is similar to the well-known expression for the dwell time derived using other approaches (for example, see [8, 24–26]) . However, the similarity is purely in appearance. We derived Eqs. (33) for transmission and reflection based on the exact solutions of the Schrödinger equation which describe both processes in all stages of scattering. In addition, Eqs. (33) specify the Larmor times; thus we have assigned them the status of measurable quantities.

Let us reveal now relations between the Larmor tunneling times described by Eq. (33) and dwell times (12) and (15). To this end, we consider, for example, the case of transmission and write down the function $\psi_{\rm tr}(x,t)$ as follows:

$$\psi_{\rm tr}(x,t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}A_{\rm in}(k)\psi_{\rm tr}(x,k)e^{-iE(k)t/\hbar}dk . \tag{34}$$

Let us transform the integral $I=\int_{-\infty}^{\infty}dt\int_a^b dx\left|\psi_{\rm tr}(x,t)\right|^2$ in Eq. (33). Taking into account Eq. (34) and integrating over t, we obtain

$$I=\frac{\hbar}{\pi}\int_{-\infty}^{\infty}dk'dkA_{\rm in}^*(k')A_{\rm in}(k)\int_a^b dx\psi_{\rm tr}^*(x,k')\psi_{\rm tr}(x,k)\times\lim_{\Delta t\to\infty}\frac{\sin\left[\left(E(k')-E(k)\right)\Delta t/\hbar\right]}{E(k')-E(k)} .$$

However (for example, see [27]),

$$\lim_{\Delta t\to\infty}\frac{\sin\left[\left(E(k')-E(k)\right)\Delta t/\hbar\right]}{E(k')-E(k)}=\frac{\pi}{\hbar}\delta\left[\left(E(k')-E(k)\right)/\hbar\right]=\frac{\pi m}{\hbar^2 k}\left[\delta(k'-k)-\delta(k'+k)\right] .$$

If the expression for I is symmetric in variables k and k', it can be easily demonstrated that contribution of $\delta(k'+k)$ to this integral is equal to zero. As a result, we have

$$\tau_{\rm tr}^{\rm L}=\frac{m}{T\hbar}\int_{-\infty}^{\infty}dk\left|A_{\rm in}(k)\right|^2k^{-1}\int_a^b\left|\psi_{\rm tr}(x,k)\right|^2dx,\quad \tau_{\rm ref}^{\rm L}=\frac{m}{R\hbar}\int_{-\infty}^{\infty}dk\left|A_{\rm in}(k)\right|^2k^{-1}\int_a^{x_c}\left|\psi_{\rm ref}(x,k)\right|^2dx$$

(we note that the integrands have no singularity at the point $k=0$). Furthermore, taking into account Eqs. (12) and (15) and the relation $\psi(x,-k)=\psi^*(x,k)$, we finally obtain

$$\tau_{\rm tr}^{\rm L}=\frac{1}{T}\int_0^{\infty}\varpi(k)T(k)\tau_{\rm dwell}^{\rm tr}(k)dk,\quad \tau_{\rm ref}^{\rm L}=\frac{1}{R}\int_0^{\infty}\varpi(k)R(k)\tau_{\rm dwell}^{\rm ref}(k)dk , \tag{35}$$

where $\varpi(k) = |A_{\mathrm{in}}(k)|^2 - |A_{\mathrm{in}}(-k)|^2$.

We note that in the case of passage of the particle under the wide rectangular potential barrier, $|\tau_0| << \tau^{\mathrm{L}}_{\mathrm{ref}} << \tau^{\mathrm{L}}_{\mathrm{tr}}$ [see Eqs. (13), (16), and (28)]. In this case, measurements of the precession angles $\Delta\phi^{(\infty)}_{\mathrm{tr}}$ and $\Delta\phi^{(\infty)}_{\mathrm{ref}}$, where $\Delta\phi^{(\infty)}_{\mathrm{tr}} = \omega_{\mathrm{L}}(\tau_0 + \tau^{\mathrm{L}}_{\mathrm{tr}})$ and $\Delta\phi^{(\infty)}_{\mathrm{ref}} = \omega_{\mathrm{L}}(-\tau_0 + \tau^{\mathrm{L}}_{\mathrm{ref}})$, for the transmitted and reflected particles, respectively, give the direct method of measuring the times $\tau^{\mathrm{L}}_{\mathrm{tr}}$ and $\tau^{\mathrm{L}}_{\mathrm{ref}}$.

CONCLUSIONS

Thus, according to our approach, scattering of the particle on the one-dimensional potential barrier is the combined process involving two matched alternative processes of transmission and reflection. On the example of symmetric potential barriers we have demonstrated that the state of the entire quantum ensemble of particles for the given problem can be unambiguously represented as a superposition of two states describing the sub-ensembles of the transmitted and reflected particles in all stages of scattering.

For both processes, we have introduced the group (local and asymptotic) tunneling times, dwell time, and Larmor tunneling time. The proposed definitions of the tunneling times differ significantly from their well-known analogs, and this is most vividly illustrated by the example of particle tunneling through a wide rectangular barrier. In this case, all well-known concepts predict faster-than-light particle passage through the barrier region (the so-called Hartman effect). On the contrary, according to our approach, the average particle velocity in the barrier region is much smaller than outside of the barrier. Thus, our investigations actually demonstrate that in contrast with the existing notions, quantum mechanics admits the model of one-dimensional scattering free from the so-called quantum nonlocality.

We note that the local group tunneling time is suitable for estimation of the dwell time of the particle in the barrier region when the particle state is described by a not too wide wave packet. On the contrary, the dwell time is suitable for the wave packets whose widths considerably exceed the barrier width. As to the Larmor tunneling time, this characteristic is universal. It can be used for wave packets of arbitrary width. In addition, it is important to emphasize that the Larmor tunneling time can be measured and hence the approach can be verified experimentally.

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